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COMMENT

Comment on 'Bohmian prediction about a two double-slit experiment and its disagreement with standard quantum mechanics'

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Abstract

Recently Golshani and Akhavan (Golshani M and Akhavan O 2001 *J. Phys. A: Math. Gen.* **34** 5259) proposed an experiment that should be able to distinguish between standard quantum mechanics and Bohmian mechanics. It is our aim to show that the claims made by Golshani and Akhavan are unwarranted.

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1. Introduction

According to standard quantum mechanics (SQM), the complete description of a physical system is provided by its wavefunction. In Bohmian mechanics (BM)[1] this standard description of quantum phenomena, by means of the wavefunction ψ, is enlarged by considering particles that follow definite tracks in spacetime (dependent on the initial conditions). The positions of a particle on these tracks act as the hidden variables of SQM. The positions of the particles are hidden because BM is constructed in a way to give the same statistical predictions as SQM if a measurement is performed. This is accomplished by assuming the probability distribution for an ensemble in BM to be the same as the quantum mechanical distribution. This distribution is called the *quantum equilibrium distribution* (see section 2).

Yet, recently Golshani and Akhavan [2] proposed an experiment that should be able to distinguish between SQM and BM at the level of individual detections and at the statistical level. It is the aim of the present work, however, to show that the claims made by Golshani and Akhavan are unfounded. Moreover it should be clear that, with the *quantum equilibrium hypothesis* in mind, one cannot obtain a disagreement between SQM and BM. Only a modified or extended Bohmian theory can yield experimentally observable differences with SQM. This

[1] For a mathematical review see [1].

stresses once more the fact that BM is nothing more than a possible (causal) interpretation of SQM, as is clearly stated in the beginning by Bohm [3, 4].

2. Quantum equilibrium hypothesis

In SQM a physical system is described in configuration space by its wavefunction $\psi(\mathbf{x}_1, \ldots, \mathbf{x}_n, t)$, dependent on n 3-vectors $\mathbf{x}_j$. This wavefunction obeys the Schrödinger equation

$$\mathrm{i}\hbar \frac{\partial \psi(\mathbf{x}_1, \ldots, \mathbf{x}_n, t)}{\partial t} = \hat{H}\psi(\mathbf{x}_1, \ldots, \mathbf{x}_n, t). \tag{1}$$

Given an initial wavefunction $\psi(\mathbf{x}_1, \ldots, \mathbf{x}_n, 0)$ this equation can be solved to give a unique solution $\psi(\mathbf{x}_1, \ldots, \mathbf{x}_n, t)$. When a position measurement is performed on an ensemble of identically prepared systems (all described by the same wavefunction), the probability $P(\mathbf{Q}_1, \ldots, \mathbf{Q}_n, t_0)$ of making a joint detection at a certain time t_0 of the n-particles at positions $\mathbf{Q}_1, \ldots, \mathbf{Q}_n$ in physical space is given by

$$P(\mathbf{Q}_1, \ldots, \mathbf{Q}_n, t_0) = \psi^*(\mathbf{Q}_1, \ldots, \mathbf{Q}_n, t_0)\psi(\mathbf{Q}_1, \ldots, \mathbf{Q}_n, t_0). \tag{2}$$

In BM, SQM is considered as an incomplete theory. Apart from a wavefunction (obeying (1)) one introduces additional (hidden) variables to describe the physical system. These hidden variables are n vectors that have to be interpreted as *actual* position vectors $\mathbf{X}_k(t)$ associated with n particles in three-dimensional physical space. According to BM these vectors are also the position vectors revealed in a position measurement. This is contrary to SQM where particles do not exist as localized entities, i.e. as entities that have position vectors, until a position measurement is performed.

Bohm [3, 4] obtained the laws of motion for the particles by giving a new interpretation to the real and imaginary part of the Schrödinger equation. The real part is interpreted as a classical Hamilton–Jacobi equation with an additional quantum mechanical potential, the *quantum potential*. This interpretation leads to the following differential equations for the position vectors $\mathbf{X}_k(t)$:

$$\frac{\mathrm{d}\mathbf{X}_k}{\mathrm{d}t} = \frac{\hbar}{m_k} \mathrm{Im} \left. \frac{\psi^*(\mathbf{x}_1, \ldots, \mathbf{x}_n, t)\nabla_k \psi(\mathbf{x}_1, \ldots, \mathbf{x}_n, t)}{|\psi(\mathbf{x}_1, \ldots, \mathbf{x}_n, t)|^2} \right|_{\mathbf{x}_j = \mathbf{X}_j} \tag{3}$$

where m_k is the mass of the kth particle. Once we have a solution for equation (1), equation (3) can be solved given the initial positions $\mathbf{X}_k(0)$. In this way the n actual position vectors $\mathbf{X}_k(t)$ of the particles are uniquely determined. If we then consider an ensemble of systems, all described by the same wavefunction, then this ensemble determines a probability distribution $\rho(\mathbf{X}_1, \ldots, \mathbf{X}_n, t)$ of the actual position vectors of the n particles. This is the distribution that would be obtained, according to BM, when a position measurement on an ensemble was performed. If we want BM to give the same predictions as SQM in a position measurement, then the probability distribution P of SQM in equation (2), has to be the same as the probability distribution ρ of BM, i.e. we must have

$$\rho(\mathbf{X}_1, \ldots, \mathbf{X}_n, t) = |\psi(\mathbf{X}_1, \ldots, \mathbf{X}_n, t)|^2 \tag{4}$$

for all times t. If this equality is assumed, and this is what is done in BM [3, 4], the imaginary part of the Schrödinger equation

$$\frac{\partial|\psi|^2}{\partial t} + \sum_k \nabla_k \cdot (\mathbf{v}_k|\psi|^2) = 0 \tag{5}$$

with

$$\mathbf{v}_k = \frac{\hbar}{m_k} \operatorname{Im} \frac{\psi^* \nabla_k \psi}{|\psi|^2} \tag{6}$$

is the continuity equation, describing the conservation of the probability density of the particles. In fact it is sufficient that we assume

$$\rho(\mathbf{X}_1, \ldots, \mathbf{X}_n, t_0) = |\psi(\mathbf{X}_1, \ldots, \mathbf{X}_n, t_0)|^2 \tag{7}$$

at a certain time t_0 (for example, at $t_0 = 0$) because both $|\psi|^2$ and ρ satisfy the continuity equation. Thus, as far as predictions involving particle positions are concerned, BM is in complete accordance with SQM if the initial particle positions $\mathbf{X}_k(0)$ are distributed according to $|\psi(\mathbf{X}_1, \ldots, \mathbf{X}_n, 0)|^2$ in the ensemble. This is what is called the *quantum equilibrium hypothesis* (QEH) by Dürr, Goldstein and Zanghì [5] and the distribution is called the *quantum equilibrium distribution*. Because every measurement is in fact a position measurement, it is clear that there can never be an experimental difference between BM and SQM, if prior to the experiment, the Bohmian particles in the ensemble are distributed according to the QEH. This also implies that, despite definite trajectories in BM, we can only predict and verify relative frequencies. Hence, an individual event cannot be studied independently from the ensemble.

The only difference that remains between BM and SQM is an interpretational one. In BM $\rho(\mathbf{Q}_1, \ldots, \mathbf{Q}_n, t_0)$ is interpreted as the probability of the particles really *being* at the positions $\mathbf{Q}_1, \ldots, \mathbf{Q}_n$ at time t_0 whereas in SQM $P(\mathbf{Q}_1, \ldots, \mathbf{Q}_n, t_0)$ is the probability of the particles *being detected* at the positions $\mathbf{Q}_1, \ldots, \mathbf{Q}_n$ at time t_0.

One of the reasons why some physicists find it hard to accept the Bohmian interpretation of quantum mechanics is the fact that we cannot observe a particle without disturbing its movement, i.e. we cannot obtain knowledge of the position of the particle without changing its wavefunction (this is the collapse in SQM). This changing wavefunction then leads to changing particle velocities (as follows from (3)), leaving a disturbed system. The best example of this is the diffraction at a slit: the smaller the slit (i.e. the better we try to get the initial position in the slit), the wider the scattering angle. In this way a quantum mechanical measurement is very different from a measurement in classical mechanics, where trajectories of objects are generally accepted because one can infer successive positions of an object without disturbing its motion, for example, by using light that scatters from the object.

We want to remark that the QEH was already postulated by Bohm in order to assure complete equivalence between BM and SQM. So BM does not provide us with new experimentally verifiable predictions, but instead gives us a broader conceptual framework that may serve as a basis for new or modified mathematical formulations for the description of physical systems. In such theories the QEH will evidently break down and this is what Bohm meant in [4]:

> An experimental choice between these two interpretations cannot be made in a domain in which the present mathematical formulation of the quantum theory is a good approximation, but such a choice is conceivable in domains, such as those associated with dimensions of the order of 10^{-13} cm, where the extrapolation of the present theory seems to break down and where our suggested new interpretation can lead to completely different kinds of predictions.

Such modifications and extensions of BM are, for example, given by Bohm [3, 6–9].

It has also to be noted that a breakdown of the QEH would have a remarkable consequence. Valentini [10] showed that the principle of signal locality (i.e. the impossibility of practical instantaneous signalling) is valid if and only if the Bohmian probability distribution ρ, equals the quantum mechanical distribution $|\psi|^2$. Thus a violation of the QEH implies the possibility of instantaneous signalling.

3. Outline and discussion of the experiment

The proposed experiment of Golshani and Akhavan [2], makes use of a pair of identical, non-relativistic, bosonic or fermionic particles labelled 1 and 2 with total momentum zero. The particles are assumed to emerge, pair by pair (so there is only one pair of particles in the device at a time), from a point source placed in the middle between two screens. The source is taken as the origin of the x–y coordinate system. The screens have both identical slits, symmetrically around the x-axis, labelled A, A', B and B'. The coordinates of the slits are $(\pm d, \pm Y)$. For convenience we adopted here, and in the following, the notation of [2].

Before we proceed with the discussion of the alleged experimental incompatibility between BM and SQM, we want to point at a technical difficulty concerning the point source emitting the particles. A point source is in general incompatible with opposite momenta, because the Heisenberg uncertainty $\Delta(x_{1i} + x_{2i})\Delta(p_{1i} + p_{2i}) \geqslant \hbar$ is valid for every Cartesian component i. However, this problem, which has been discussed in detail in [11, 12], has no consequences for the subsequent analysis of the experiment.

The detection will take place after the particles passed the slits. The wavefunction of the system describing the correlated particles emerging from the slits is taken as

$$\begin{aligned}\psi(x_1, y_1, x_2, y_2, t) = N[&\psi_A(x_1, y_1, t)\psi'_B(x_2, y_2, t) \pm \psi_A(x_2, y_2, t)\psi'_B(x_1, y_1, t)\\ &+ \psi_B(x_1, y_1, t)\psi'_A(x_2, y_2, t) \pm \psi_B(x_2, y_2, t)\psi'_A(x_1, y_1, t)]\end{aligned} \tag{8}$$

with

$$\begin{aligned}\psi_{A,B}(x, y, t) &= \left(2\pi\sigma_t^2\right)^{-1/4}\ \mathrm{e}^{-(\pm y - Y - u_y t)^2/4\sigma_0\sigma_t + \mathrm{i}[k_x(x-d) + k_y(\pm y - Y - u_y t/2) - E_x t/\hbar]}\\ \psi_{A',B'}(x, y, t) &= \left(2\pi\sigma_t^2\right)^{-1/4}\ \mathrm{e}^{-(\pm y - Y - u_y t)^2/4\sigma_0\sigma_t + \mathrm{i}[-k_x(x+d) + k_y(\pm y - Y - u_y t/2) - E_x t/\hbar]}\end{aligned} \tag{9}$$

the Gaussian waves generated by the respective slits and

$$\sigma_t = \sigma_0\left(1 + \frac{\mathrm{i}\hbar t}{2m\sigma_0^2}\right) \qquad u_{x,y} = \frac{\hbar k_{x,y}}{m} \qquad E = \frac{1}{2}mu_x^2. \tag{10}$$

The upper sign in (9) describes the bosonic case and the lower sign the fermionic case.

The detections of the particles takes place on two screens, S_1 and S_2, placed at a distance $d + D$ from the source, parallel with the y-axis. Only pairs of particles which arrive simultaneously will be considered in the experiment. Therefore, the motion in the x-direction is irrelevant and only the motion in the y-direction will be taken into account. If we suppose the detectors to be idealized pointdetectors, then SQM gives the following probability for detecting the pair of particles (in the ensemble) at time t_0 at positions $y_1 = Q_1$ and $y_2 = Q_2$ on the two screens:

$$P(Q_1, Q_2, t_0) = |\psi(y_1, y_2, t)|^2|_{y_1=Q_1, y_2=Q_2, t=t_0} = |\psi(Q_1, Q_2, t_0)|^2. \tag{11}$$

If the detectors detect over regions ΔQ_1 and ΔQ_2 then the probability of joint detection is given by

$$\bar{P}_{12} = \int_{Q_1}^{Q_1+\Delta Q_1}\int_{Q_2}^{Q_2+\Delta Q_2} \mathrm{d}y_1\,\mathrm{d}y_2\,|\psi(y_1, y_2, t)|^2. \tag{12}$$

According to the QEH these probabilities must be the same for BM.

The proposed discrepancy between BM and SQM is based on the following observation. If we take $y_1(t)$ and $y_2(t)$ as the y-components of the Bohmian trajectories of the particles and define $y(t)$ as the centre of mass in the y-direction

$$y(t) = \tfrac{1}{2}(y_1(t) + y_2(t)) \tag{13}$$

then one finds by using (3), (8) and (9) that

$$y(t) = y(0)\sqrt{1 + \left(\hbar/2m\sigma_0^2\right)^2 t^2}. \tag{14}$$

If at $t = 0$ the centre of mass of the particles is exactly on the x-axis (i.e. $y(0) = 0$) then the centre of mass will always remain on the x-axis. So according to BM the particles will always be detected symmetrically about the x-axis if $y(0) = 0$ for each pair of particles. Golshani and Akhavan wrongly assume that the relation $y(0) = 0$ is satisfied for every pair of particles. The symmetrical Bohmian prediction would then be in contradiction with possible assymmetric detections predicted by SQM (12). But it is clear that this reasoning involves a violation of the QEH, $y(0)$ is distributed according to the absolute square of the wavefunction and is unknown in principle without measurement.

According to Golshani and Akhavan the final interference pattern predicted by BM is the same as the pattern predicted by SQM, therefore they also consider selective detections, which would result in an incompatibility between BM and SQM at the ensemble level. They consider two cases. In the first case they assume $y(0) = 0$ for every pair of particles. If they only record particles that arrive on the upper half of the screen S_1, then the corresponding (Bohmian) particles will be detected on the lower half of the screen S_2. This result would be incompatible with the possible detection of both particles on the upper halves of S_1 and S_2, predicted by SQM. Again the discrepancy is based on a false assumption about the initial Bohmian positions. However, in the second case they consider a distribution for $y(0)$ according to the QEH. This would not alter the previous result, when some particular conditions are taken into account. Therefore they assume that $\langle y(0)\rangle = 0$ and small values for $\Delta y(0)$ and $\hbar t/2m\sigma_0^2$. Now remark that the first two conditions imply that $y(0)$ is very small, so we are in fact dealing with the first case. In this case not only BM will again predict symmetrical detections, but also SQM will, because the third condition implies slow spreading Gaussians in (9). So $\langle y(t)\rangle$ and $\Delta y(t)$ will remain small, which implies a symmetrical prediction for SQM as well.

4. Conclusion

The cause of the different predictions made by BM and SQM in Golshani and Akhavan's proposal is that for the Bohmian description of the system, a special assumption is made concerning the initial conditions. This assumption concerns a restriction on the initial positions of the Bohmian particles. Because the initial positions of the particles have a distribution that is the same in both quantum mechanics and Bohmian mechanics, the additional assumption on the initial particle positions is unjustified. This implies that experimental discrepancies at both the individual level and the ensemble level cannot be obtained.

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